# PRE-SERVICE TEACHERS' REFLECTIONS OF THEIR IMPLEMENTATION OF MATHEMATICAL TEACHING PRACTICES

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## Abstract

In this qualitative research study, preservice teachers were asked to reflect on their implementation of effective mathematical teaching practices as outlined by the National Council of Teachers of Mathematics. The responses and reflections specifically gauged preservice teachers' strengths and areas of growth relating to effectiveness in teaching mathematics in both face-to-face and synchronous online lessons. Participants in this study were most likely to identify practices involving setting goals and assessing student understanding as strengths. Supporting productive struggle and posing questions were the most common practices selected as areas for improvement. The findings from this study promote the use of mathematical teaching practices to promote deep reflection of preservice teachers as a tool to improve their overall teaching. Faculty of Educator Preparation Programs (EPP) may find this study useful when exploring ways to improve reflection and teaching of preservice teachers.

Keywords: *mathematical teaching practices, preservice teachers, reflection, online teaching*

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The purpose of this qualitative study was to examine the reflections of preservice teachers after teaching a mathematics lesson to their university peers in a mathematics methods course in their EPP. Participants in the first semester of data collection delivered traditional face-to-face lessons while participants in the second semester delivered synchronous online lessons. Elementary and middle level preservice teachers (PSTs) answered several lesson reflection prompts including ones regarding the implementation of the National Council of Teachers of Mathematics (NCTM) (2014) effective mathematics teaching practices (MTPs). The study was led by the following research questions:

1. What do elementary and middle level PSTs believe are their mathematics teaching strengths and weaknesses relating to the MTPs?
2. How does the format (face-to-face or synchronous online) of lessons impact the implementation of MTPs?

## Literature Review

Some members of society still believe mathematics instruction should take a traditional approach which emphasizes rote memorization, use of standard algorithms, and instruction that is teacher-centered. The NCTM (2000, 2014) describes those views as unproductive, and believes student-centered



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constructivist approaches that encourage problem solving should be used to teach and do mathematics. To promote deep learning, the NCTM (2014) developed a framework of eight MTPs. These practices are:

1. Establish mathematics goals to focus learning
2. Implement tasks that promote reasoning and problem solving
3. Use and connect mathematics representations
4. Facilitate meaningful mathematical discourse
5. Pose purposeful questions
6. Build procedural fluency from conceptual understanding
7. Support productive struggle in learning mathematics
8. Elicit and use evidence of student thinking

When implementing the MTPs in mathematics instruction and curriculum, Smith et al. (2017, 2018) explained the integration of the practices in a coherent way is what truly impacts the effectiveness of a lesson. For example, Smith and colleagues (2017, 2018) pointed out questioning, productive struggle, mathematical representation, and evidence of student thinking as critical parts of the practice mathematical discourse, emphasizing the connection between MTPs and that good teaching reflects more than one practice. Smith et al. (2018) also encouraged teacher collaboration when using the MTPs as a framework to design and evaluate lessons.

The existing literature regarding the implementation of the MTPs focuses on the identification of those practices in teaching vignettes rather than participants reflecting on their own teaching (Shelton et al., 2020; Shelton et al., 2021; Wilkerson et al., 2018). Prior to evaluating the vignettes, participants were familiar with the MTPs, but when evaluating vignettes both teachers (Shelton et al., 2020) and PSTs (Wilkerson et al., 2018) described learning more about the MTPs and how they could be implemented. According to Shelton et al. (2020), participants found the vignettes helpful in reflecting on their own practices and implementation of the MTPs. However, Shelton and colleagues (2021) found PSTs held many misconceptions and sometimes shallow understandings of the MTPs when evaluating their implementation in teaching vignettes. Addressing the gap in the literature, our study examines the reflections of PSTs regarding their own teaching practices in lieu of utilizing vignettes.

## Methods

In this qualitative study, selected response and open-ended survey items were given to encourage PSTs to reflect on a mathematics lesson they created and taught. Participants designed K-8 mathematics lessons and then taught the lessons in their college mathematics methods course to their university peers. Most participants worked in small collaborative groups to design and teach their lessons. After teaching their lessons, participants independently completed a survey regarding their instruction and implementation of the MTPs. The participants were elementary and middle level senior education majors enrolled in an undergraduate mathematics methods course which is taken the semester prior to clinical teaching.

For this study, data was collected from two semesters and included 70 participants. More than eighty percent of participants were elementary education majors (n=57), about 19% were mid-level education majors (n=13), and one participant was a K-12 special education major. More than 90% of participants identified as women (n=65) and the rest identified as men (n=5). The COVID - 19 pandemic occurred during data collection, which impacted the format of the lessons taught. PSTs participating during the first semester of data collection taught face-to-face lessons (n=30) to their peers, while PSTs in



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the second semester of data collection taught remote synchronous online lessons (n=40) to their peers via Zoom.

PSTs were asked various questions to prompt deep reflection of their lessons and instruction. For this study, only three prompts from the reflection assignment were analyzed. PSTs were asked to identify the MTPs (NCTM, 2014) implemented in their lessons, explain how they did so, and to describe the teaching practices they wished to improve upon. During analysis the survey response items were coded based on the eight MTPs reflecting a provisional coding (Saldaña, 2013) approach.

## Findings

Participants were asked to complete both selected-response and open-ended prompts regarding the MTPs. The PSTs selected which of the MTPs they believed they achieved in the lesson they designed and taught to their peers in their mathematics methods course. Participants were also asked to explain how they believed they met those practices. In addition, PSTs identified which of the practices they hoped to improve upon. The qualitative analysis of the findings is organized by each of the MTPs and how participants reflected on their implementation of each. The summary of successfully implemented MTPs by PSTs during their teaching experience is reported in Table 1. The summary of MTPs participants identified as an area for growth is reported in Table 2.

### Goals

Most participants reported meeting the first mathematics teaching practice, “establish mathematics goals to focus learning.” Specifically, 80% of the PSTs teaching remote lessons and about 67% of PSTs teaching lessons face-to-face felt they achieved this practice. A few participants mentioned they began their lesson planning with identifying a state standard, “We built our lesson through backwards design by picking our standards, determining our driving questions, and then determining goals and objectives before going into the step-by-step of the lesson.” Several PSTs discussed writing objectives like “ABCD objectives” and emphasized alignment, “we wrote goals and objectives that aligned to our [state] standards.”

Only 3% of participants teaching face-to-face lessons and 12.5% of participants teaching synchronous remote lessons identified establishing goals as an area for improvement. A participant teaching an online synchronous lesson believed stated, “we should've written or stated our learning objective for the lesson before beginning to make sure students were focused on the task.” One middle grades PST who taught face-to-face thought this practice was a challenge due to the inquiry format of her lesson:

> I don’t think the instructional goals for this lesson were clear. I think because I wanted them to discover the concepts of volume, I didn't want to give too much away but I should have given more guidance. I should have clearly stated that the students were working in a group to discover the formula for the volume of the cone, which would allow them to mathematically prove the number of candies in the cone.

### Reasoning and Problem Solving

Ninety percent of participants teaching face-to-face and 75% of those teaching synchronously online claimed to “implement tasks that promote reasoning and problem solving.” When explaining how



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they met this practice, some participants specifically described a problem from their lesson. For example, a PST who taught face-to-face discussed:

> Basically, we were telling them what to do, but not how to do it. We promoted reasoning and problem solving by giving students a 'problem' which was, they need[ed] to plan a baby shower for the teacher on a budget but had to have enough stuff to feed everyone and buy a gift. The students had to reason with each other to determine the budget.

A few PSTs encouraged multiple approaches to solving the same problem, "each student used their own approach to estimate the number of candies in the cone. Several methods were discussed within their group when trying to determine the best ways to gather data." When describing how they promoted reasoning and problem solving, participants described students "discovering" concepts, using "estimation" and "asking questions."

When asked which MTPs they could improve upon, only about 7 % of both groups emphasized reasoning and problem solving. Participants expressed that if they could change their lesson, they would have created problems that were "more challenging" and "interesting to the students." One participant believed their selected instructional model limited student problem solving, "I really like when students discover their learning and I wish that we could have implemented more of this. However, with direct instruction, I do not think it would have worked."

## Mathematical Representations

The teaching practice, "use and connect mathematical representations," was implemented by about 43% of PSTs teaching face-to-face and almost 60 % of PSTs teaching remote lessons. Several participants emphasized the use of manipulatives and tools like scales, play-doh, shapes, and money as types of mathematical representations. Participants in both groups also mentioned the use of real-world problems, tables, charts, and graphs as representations to deepen understanding of concepts. One participant described how they used multiple types of representation, "We connected the terms even and odd to more 'fun, relatable' terms like 'odd man out' and 'buddy.' We created a task that let the students explore on their own with the number chart and online manipulatives!"

Ten percent of participants teaching face-to-face as well as ten percent of those teaching remote lessons identified this practice as an area of improvement. When describing how they could improve their lessons, a few participants mentioned incorporating student use of "manipulatives" and connecting concepts to the "real-world." Two PSTs mentioned not achieving this practice well due to restrictions beyond their control like, "only having twenty minutes or so was not enough time to accurately use and connect mathematical practices…It would need to be done over several days with students gathering and dissecting the data" and "our standard seemed easy, but it was not. It was very limited on how we could teach it."

## Mathematical Discourse

Forty percent of face-to-face teaching and 50% of remote teaching participants reported achieving the practice, "facilitating meaningful mathematical discourse." Several PSTs described their questioning to promote discourse and discussion among their students. A particular PST pointed out several ways they facilitated math discourse, "My co-teachers and I promoted meaningful mathematical discourse by posing pointed questions. As I walked around, I asked the students to tell me their predictions. I then used 'talk moves' to ask students to explain their reasoning." Some participants provided little detail when



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describing how they met this practice, “the project we had with students do with the skittles and the graphs helped us facilitate meaningful mathematical discourse.”

Ten percent of remote teaching participants and about 7% of face-to-face teaching participants described the practice “facilitate meaningful mathematical discourse” as a weakness. One participant felt they could have encouraged better discourse by encouraging and modeling the use of mathematical vocabulary. Another participant described facilitating discourse early in their lesson but felt that “it could have been more meaningful if we would’ve allowed them to have the same opportunity near the end of the lesson.” A couple PSTs who taught remote online lessons used break-out rooms for small group discussions, and one pointed out a change they would like to make, “I think when we [class] went into breakout rooms we [teachers] should have gone too just to observe the discussion. In an actual classroom, we would have gotten to hear what our students were discussing.”

**Questioning**

When asked if they implemented the practice, “pose purposeful questions,” almost 75% of face-to-face teaching participants responded yes, compared to 50% of participants teaching online. PSTs described using questioning in various stages of the lesson and for the purpose of “scaffold[ing],” “activating prior knowledge,” “check[ing] for understanding,” and “guiding,” students to discover the content. Participants also described posing questions to both groups of students and to students individually.

As an area for improvement, about 27% of participants who taught face-to-face and 20% of participants who taught remote lessons chose “pose purposeful questions.” Some PSTs reflected that their better use of questioning would have “improved student understanding” and allowed “student discussions to be more in-depth.” While most agreed they did pose questions, they argued they could have done better. One PST explained:

> I wish that I would have been more effective in posing purposeful questions in the last few steps of this model. I felt like it was difficult for me to think of questions to ask without explicitly telling students the answers. To be more effective next time, I could have questions thought of/wrote down to refer to.

Two participants pointed out the challenge of posing difficult questions when teaching elementary lessons to their college peers:

> This lesson is for second graders, so it was a little harder to do this with college students because we obviously know the difference between even and odd, but second graders don’t. So, I think we would’ve had a better discussion with younger students.

**Procedural Fluency**

More than 35% of participants teaching face-to-face and 50% of participants teaching remotely reported using the practice, “build procedural fluency from conceptual understanding.” One PST described, “we used models to support the students’ understanding of our concept as well as discussing and explaining” as methods of building procedural fluency from conceptual knowledge. Another participant shared, “through creating general rules with the class at the end of the lesson, I was able to build procedural fluency using the conceptual understanding of our class.”



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Only 10% of participants who taught face-to-face lessons and 7.5 % of students who taught remote lessons identified "build procedural fluency from conceptual understanding" as a mathematical practice they hoped to improve upon. Multiple PSTs argued lack of time as a reason for struggling with building procedural fluency, "if we had more time, we could have included more examples and emphasized the pattern…" One participant who taught via Zoom, felt more teacher explanation could have built procedural fluency, "I wish I would have explained step-by-step instructions on how to draw different types of angles with protractors."

## Productive Struggle

About 33% of participants teaching face-to-face and 20% of those who taught remote online lessons claimed to, "support productive struggle in learning mathematics." The few participants who believed they addressed this practice, provided specific examples from their lessons. According to one PST who taught face-to-face, "my co-teachers and I supported productive struggle by supplying the 'uh oh cards' which challenged students' to adjust and modify their plan and/or solution." Another participant discussed creating struggle by not allowing the use of calculators. A few participants required students to explain thinking and refrained from giving answers and one participant who taught online explained, "we also encouraged and praised all interactions without confirming or correcting student assumptions." One person described using logic to promote productive struggle, "Joe answered one of our problems incorrectly and we were able to take his answer and show him why it wouldn't be that."

Supporting productive struggle was chosen as an area for improvement by 20% of participants teaching face-to-face and 15% of participants teaching remote lessons. Limited time and teaching adults' elementary concepts were common reasons for not implementing this practice effectively. One participant shared, "I don't feel like the students had to struggle too much with our mathematical learning concept, but I might feel differently if it was a real kindergarten class." Another PST described the challenge of letting students struggle:

> When my classmates had a question about how they were supposed to model their thinking, my first instinct was to immediately tell them the answer I was looking for. Instead of giving the answers, my group could have worked together to create questions that would have provoked our classmates to think more deeply to create a solution to the problem they were given.

## Evidence of Student Thinking

Regarding the practice, "elicit and use evidence of student thinking," about 77% of face-to-face teaching participants and 68% of online teaching participants reported demonstration during their lessons. Participants referenced "assessments" and "activities to gauge student thinking" when discussing this practice. Formative assessments like requiring students to explain their thinking, reflection questions, exit tickets, digital quizzes, student presentations, and games were mentioned. One online teaching participant alluded to using student assessment data to make decisions, "we had a plan in place if a student did not understand the concepts we were covering."

Only one participant teaching a remote lesson and one participant teaching face-to-face lesson commented "elicit and use evidence of student thinking" as an area of weakness. One participant reflected more discussion and "why" questions would have resulted in more student thinking evidence. The other PST expressed this practice as a challenge when working with college students, "I think that when



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teaching actual students, the lesson will flow more organically without struggling for engagement and ability to cater the lesson."

**Table 1**

*Self-Reported PSTs Successful Implementation of the MTPs via Face-to-Face vs. Synchronous Online Instruction*

| Mathematical Teaching Practice | Face-to-Face (n=30) | | Online (n=40) | |
|---|---|---|---|---|
| | n | % | n | % |
| Establish Mathematics Goals to Focus Learning | 20 | 66.7 | 32 | 80 |
| Implement Tasks that Promote Reasoning and Problem Solving | 27 | 90 | 30 | 75 |
| Use and Connect Mathematical Representations | 13 | 43.3 | 23 | 57.5 |
| Facilitate Meaningful Mathematical Discourse | 12 | 40 | 20 | 50 |
| Pose Purposeful Questions | 22 | 73.3 | 20 | 50 |
| Building Procedural Fluency from Conceptual Understanding | 11 | 36.7 | 20 | 50 |
| Support Productive Struggle in Learning Mathematics | 10 | 33.3 | 8 | 20 |
| Elicit and Use Evidence of Student Thinking | 23 | 76.7 | 27 | 67.5 |



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**Table 2**

*Self-Reported PSTs Improvement Areas of the MTPs via Face-to-Face vs. Synchronous Online Instruction*

| Mathematical Teaching Practice | Face-to-Face (n=30) | | Online (n=40) | |
|---|---|---|---|---|
| | n | % | n | % |
| Establish Mathematics Goals to Focus Learning | 1 | 3.3 | 5 | 12.5 |
| Implement Tasks that Promote Reasoning and Problem Solving | 2 | 6.7 | 3 | 7.5 |
| Use and Connect Mathematical Representations | 3 | 10 | 4 | 10 |
| Facilitate Meaningful Mathematical Discourse | 2 | 6.7 | 4 | 10 |
| Pose Purposeful Questions | 8 | 26.7 | 8 | 20 |
| Building Procedural Fluency from Conceptual Understanding | 3 | 10 | 3 | 7.5 |
| Support Productive Struggle in Learning Mathematics | 6 | 20 | 6 | 15 |
| Elicit and Use Evidence of Student Thinking | 1 | 3.3 | 1 | 2.5 |

## Discussion

This study was led by two research questions. The first focused on the perceived teaching strengths and weaknesses of elementary and middle level PSTs. Participants chose a variety of MTPs they believed they implemented successfully. Most participants identified the first and last practices as strengths when teaching their lessons. The first practice focuses on establishing mathematics goals while the last emphasizes assessing student understanding. The reason the participants felt confident in these two areas could be attributed to the emphasis of backward design (Wiggins & McTighe, 2005) throughout their teacher education program, as well as specific locations for state standards, objectives, and assessments on their required lesson plan template. The majority of participants in both teaching groups also felt they implemented tasks that promote reasoning and problem solving. Despite the delivery format of the lesson, participants successfully incorporated mathematical tasks in turn creating student-centered lessons that required students to solve problems. Participants did not just lecture and send students away to do independent practice, they provided opportunities for students to solve problems during the lesson. Like the findings from Shelton et al. (2021), participants sometimes provided very little evidence when explaining how they implemented the MTPs which could indicate incomplete understanding of the MTPs or lack of reflection skills.

All the practices were chosen as a weakness or an area of improvement by at least one participant. However, supporting productive struggle and posing purposeful questions were the most identified



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weaknesses. Participants highlighted the need to plan deep thinking questions to promote productive struggle, mirroring the idea that MTPs should be integrated in a coherent way to enhance effectiveness of lessons (Smith et al., 2017; 2018). Several participants also mentioned the difficulty of creating productive struggle when teaching an early elementary lesson to their college peers, emphasizing the value of K-12 field experience when teaching mathematics lessons and reflecting on their success. Regardless of teaching format, participants needed more help in posing questions and promoting productive struggle. Mathematics methods instructors and EPPs should place more emphasis on preparing PSTs in these areas and provide more opportunities to reflect on the implementation of these MTPs. To address these areas of improvement, EPPs can incorporate reflective teaching practices within seminar discussions and through early field experience. Students should have the opportunity to share their individual reflections to gauge a further discussion amongst the group of PSTs in the EPP.

Due to the COVID-19 pandemic, some participants taught lessons traditionally face-to-face while others taught in a synchronous online format. Participants who taught face-to-face were more likely to identify reasoning and problem solving, questioning, productive struggle, and using evidence of student thinking as strengths than the participants who taught online synchronous lessons. In the traditional classroom setting, PSTs might have been able to witness more student problem solving and struggle through observation and small group discussions in turn prompting more questions by the teacher, which can be more difficult in an online setting. For example, one PST who taught an online lesson, reflected that she should have joined the breakout rooms when small groups were discussing, just like she would have walked around the classroom during face-to-face small group discussions. Therefore, PSTs need more instruction on how to assess learning and use questioning effectively when teaching online.

PSTs who taught synchronous online lessons chose using mathematical representations, facilitating mathematical discourse, and building procedural fluency as MTPs strengths more often than those teaching face-to-face lessons. One possible reason for feeling more confident in using mathematical representations is that PSTs teaching online had to be more prepared when choosing manipulatives, representations, and tools. PSTs who taught face-to-face knew they had a variety of tools in their university classroom, while the online participants had to consider what resources their classmates had in their homes or what manipulatives were available online prior to the lesson.

One surprising difference was participants teaching online were more likely to choose establish goals as a weakness than their peers teaching face-to-face. While the reasons are unclear, it could be due to the stress of hosting a web session at the start of the lesson causing the PSTs to be less clear when providing goals and directions. Despite the two formats, PSTs identified a variety of strengths and weaknesses. Even when describing the challenges of teaching online, PSTs transferred their previous experience and knowledge of teaching face-to-face lessons to deliver student-centered instruction. The differences found among the two groups could also be attributed to other factors like previous experience, teaching partners, and prior knowledge.

## Conclusion

Examining PSTs' uses of MTPs promotes teacher reflection by allowing teachers to reflect on specific teaching skills. PSTs often lack depth in their reflections of their planning and teaching (Chikiwa & Graven, 2021), and using the MTPs as a guide during reflection provided much needed scaffolding for PSTs. By using specific teaching practices, participants reflected more on their lessons and instruction and less often on factors outside of their control. Due to the qualitative nature of this study, the findings are



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not generalizable, but mathematics educators, university supervisors, and EPP providers may notice similarities among their PSTs and find “transferability” when reviewing this study (Lincoln & Guba, 1985).

Furthermore, this allows teacher educators to adjust curriculum and instruction to model and encourage content specific teaching practices. The participants in this study had very little knowledge of the MTPs prior to planning and delivering their lessons, which may have limited their implementation and reflections. Mathematics methods instructors should consider introducing the MTPs early, promote the use of MTPs when PSTs are planning lessons and encourage PSTs to reflect on their implementation of those MTPs. Even though the MTPs are intertwined, it may be more beneficial to focus on just one or two of the MTPs at a time, allowing PSTs to focus their skills and build proficiency. Shelton et al. (2020, 2021) and Wilkerson et al. (2018) found participants benefited from exploring the implementation of MTPs in teaching vignettes, so PSTs may also find value in evaluating the MTPs implementation in lessons taught by their peers, cooperating teachers, and university faculty.

Using PST reflection, teacher educators can reflect and adjust their own teaching practices that benefit student engagement and understanding. With the use of continuous reflective practice, instructors can analyze instructional teaching practices that are supported by student feedback. When this process becomes habitual in practice, educators can adjust instruction to benefit from current classroom scenarios efficiently. By creating a continuous reflective practice which is guided by PSTs, instructors are better equipped to promote and support ongoing instructional improvement.

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